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A NETWORK MODEL OF SYSTEMIC RISK: STRESS TESTING THE BANKING SYSTEM[1]

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SUMMARY

Although there are many definitions of systemic risk, most agree that it manifests itself by an initial shock that results in the failure of one or more banks and then spreads out to the entire system by a contagion mechanism which can result in the failure of more banks in the system. Assuming that bank failures in the initial shock are randomly dependent on the failure probabilities of the individual banks and that the ensuing contagion process is deterministic, depending on interbank exposures, in this paper we propose a network model to analyse systemic risk in the banking system that, in contrast to other proposed models, seeks to obtain the probability distribution of losses for the financial system resulting from the shock/contagion process.

Thus, calculating the probabilities of joint failures by simulation and assuming that the matrix of bilateral interbank exposures is known, we represent systemic risk in the financial system by means of a graph and use discrete modelling techniques to characterize the dynamics of contagion and corresponding losses within the network. The probability distribution of losses, *risk profile for the Mexican banking system*, is obtained through an efficient, complete enumeration procedure of all possible bank default events in the system. This, in turn, allows the use of the wide variety of well-established risk measures to describe the fragility of the financial system. Additionally, the model allows us to perform stress tests along both the bank default probabilities and the interbank exposures and is used to assess the risk of the Mexican banking system. Copyright © 2009 John Wiley & Sons, Ltd.

1. INTRODUCTION

Systemic risk is a subject of paramount importance for regulators responsible for financial stability, but its measurement poses a formidable technical problem. Part of the difficulty is that the initial shock which causes the failure of one or more banks, and then spreads out to the entire system, can arise from a wide variety of sources, e.g. default in large payment systems or as counter party of a contingent claim of a derivatives contract in the interbank lending market. Another important difficulty is how to associate a risk measure to the contagion process itself. That is, once the initial event occurs, what is the impact on the financial system provoked by the ensuing contagion process due to banks' exposures to each other? Whereas measures such as value-at-risk (VaR), Tail-VaR and stress tests have been developed for market and credit risk, no comparable measures have been developed for systemic risk. This makes it difficult for financial authorities to design regulation that specifically addresses systemic-risk-related issues in an efficient way. A case in point is deposit insurance, the cost of which must not only contemplate the individual probabilities of bank failures,

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but also the contagion capability that particular banks have on the entire system. Thus, financial contagion is an integral part of systemic risk and cannot be disassociated from it.

In our study we employ a network model to study systemic risk and capture both the initial random shock and the ensuing contagion process. The Systemic Risk Network Model permits the estimation of the distribution of losses for the financial system due to the initial shock and the contagion process, to perform some stress tests and develop a measure of financial fragility.

The paper is organized as follows. In Section 2 we begin by reviewing the literature we consider relevant to our work, both on systemic risk and financial contagion. We then provide a brief summary of the main approaches proposed to study the phenomenon. After a brief mention of some applications of graphs and networks to economic and financial problems, we then discuss what, in our opinion, is the most relevant work on financial contagion using graph theoretical and network models, as they relate to our particular approach.

Section 3 deals with the details of our network model to study systemic risk. We explain how the proposed model captures the relationship between banks through interbank loans and how the dynamics of the contagion mechanism are characterized using discrete modelling techniques. By incorporating the individual failure probabilities of the banks in the system (assuming independence), we show how to obtain the distribution of losses in the financial system due to the initial shock and contagion through an efficient, complete enumeration procedure.

Section 4 presents an initial proposal of a measure of financial fragility for the banking system. In Section 5 we use the model to perform stress tests of the Mexican banking system. After providing full details of the enumeration procedure, we go on to show how the model can be used for stress testing the financial system and explain the experiments performed, the data used and the results obtained. Finally, in Section 6, we summarize our findings and propose possible lines of further research.

2. SYSTEMIC RISK AND FINANCIAL CONTAGION

The importance of systemic risk is its link with financial stability. In fact, one implies the other to the point that, in order to ensure financial stability, it is necessary to measure and 'manage things' so that the risk of occurrence of events that could lead to systemic crises can be avoided or mitigated. Although much has been written on systemic risk, like in De Bandt and Hartmann (2000),[2] and there is a good idea of what systemic risk is about, there is no precise, widely accepted definition, nor is there such a thing as an accepted analytical framework. The dominant idea in any definition is that systemic risk has to do with 'the risk of experiencing an event that will affect the well-functioning of the entire financial system'.[3] It is interesting to note, however, that alternative definitions also refer to the nature of the event and the mechanism of propagation that could affect the financial system. For example, contagion could occur through failures in payment systems, counter party defaults in derivatives contracts, defaults in interbank loans or a combination of these. As to the nature of the event that could cause a widespread failure, two are readily apparent:

[2] De Bandt and Hartmann provide a useful survey of published research on systemic risk up to the year 2000.

[3] For example, Kaufman (1995a: 47) defines it as 'the probability that cumulative losses will accrue from an event that sets in motion a series of successive losses along a chain of institutions or markets comprising a system. . . . That is, systemic risk is the risk of a chain reaction of falling interconnected dominos'. According to De Bandt and Hartmann (2000): 'Systemic risk (in the narrow and broad sense) can then be defined as the risk of experiencing systemic events in the strong sense'.

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(i) a shock that causes a severe dysfunctionality in a group of financial institutions or (ii) the failure of a certain number of financial institutions, either of which is transmitted to the entire financial system through one of the mechanisms previously mentioned.

Particularly noteworthy is the definition given by the Bank for International Settlements in its annual report of 1993–1994: ‘Systemic risk is the risk that the failure of a participant to meet its contractual obligations may in turn cause other participants to default, with the chain reaction leading to broader financial difficulties’ (BIS, 1994). This definition of systemic risk highlights the role of financial contagion in a systemic crisis. From this discussion one can infer that systemic risk has two components: (i) an event that causes the failure or dysfunctionality of a critical number of market participants and (ii) a contagion mechanism which propagates the failure and/or dysfunctionality to a broader number of participants or the entire system.

2.1. The Role of Financial Contagion in Systemic Risk

That financial contagion is a real threat is evidenced by the financial crises of varying degrees of severity and detonated from different sources that have been experienced in several countries in the last two decades. The savings and loans crisis in the USA in the late 1980s and early 1990s, the Mexican crisis (‘tequila effect’) of 1994–1996 and the Russian (‘long-term capital’) and Asian crises at the end of the century are among the most notable and are still fresh in our memories. Furthermore, the globalization of the financial system resulted in cross-border spillover effects that only a few years earlier would have been inconceivable and highly unlikely. It is thus important to understand the causes, the mechanics and the consequences of financial contagion, which is not an easy task. The complex way in which today’s financial institutions are related to each other make it difficult to understand conceptually and verify empirically the different sources and nature of possible destabilizing events with the ensuing contagion process and its consequences. Although the types of shock to which financial systems are particularly sensitive are fairly evident, it is not clear what causes contagion between financial institutions. From herding behaviour to sunspots, all sorts of explanations have been provided, none of which is either totally inclusive or conclusive.

Although much has been written on systemic risk and contagion, in what follows we only mention those references that provided the conceptual framework for the study of systemic risk due to financial contagion as they relate to our approach. Rochet and Tirole (1996) do not deal specifically with financial contagion, but they provide a theoretical framework to investigate interbank lending and systemic risk; they arrive at the important conclusion that, in an environment of market discipline, interbank lending could be beneficial for prudential control. Of the empirical studies on financial contagion, the most cited are Furfine (1999a,b). In his study of contagion in US banks, bank failures of ‘significant’ banks are simulated and the effect on the remaining banks is measured by estimating the expected loss in each case. Furfine acknowledges that he underestimates the size of the interbank market, as he only uses interbank federal funds exposures for his study. There are a number of papers that analyse contagion in different countries along the same lines, e.g. Sheldon and Maurer (1998) and Muller (2006) for Switzerland, Wells (2002) for the United Kingdom, Blavarg and Nimander (2002) for Sweden, Upper and Worms (2004) for Germany, Degryse and Nguyen (2004) for Belgium, Graf *et al.* (2005) for Mexico and Boss *et al.* (2004a) and Elsinger *et al.* (2006) for Austria. In later research there is a conscious effort to compensate for the underestimation of losses by considering all interbank exposures. The difficulty here is in the data, which is vague on how these are distributed. In order to deal with this problem, and with the exception of Graf *et al.* (2005) and Muller (2006), the lack of counter party information is dealt with with a

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maximum entropy algorithm in order to fill the information gaps. Another interesting approach is to use market data on the movements of stock prices, interest rates and exchange rates to infer statistically whether or not contagion occurred. In Gropp *et al.* (2006), the authors analyse contagion using distance to default measures for European banks and find evidence of cross-border contagion in Europe.

In Upper (2007), besides summarizing the previously mentioned group of papers related to simulations of financial contagion, the author goes on to evaluate the assumptions made by other authors and discusses their use for the analysis of financial stability. In his paper, Upper clearly states: 'Going forward, more work is needed on how to attach probabilities to the individual scenarios and on the micro foundations of the models'. In the approach followed in this paper, we show how the network model permits the association of bank default probabilities to the initial shock scenarios of bank failures followed by failures due to contagion, which permits the estimation of the distribution of losses for the financial system, which can in turn be used to obtain a measure of fragility.

When applicable, graph and network models possess many advantages. Besides the fact that a vast knowledge base and analytical tools are available in this field, network formulations are highly visual and dynamic, and it is possible to gain much insight and understanding on a problem by simply examining its graphical representation. Graph theory can be traced as far back as 1763 with the paper by Euler on the solution of the 'Königsberg bridge problem'. Euler 'invented' graph theory in order to solve this puzzle. In 1758, Quesnay represented the financial funds' flows in an economy as a network and it can be considered the first financial network model. In the 20th century, first Pigou (in 1920) and later Kantorovich, Hitchcock and Koopmans used a graph representation for the minimum cost transportation problem. The final breakthrough occurred in the late 1950s and early 1960s, with the work by Dantzig, Ford and Fulkerson, which paved the way for the development of a host of efficient algorithms to solve network flow problems. Applications of graph and network models in economics and finance range from currency translation to the portfolio optimization problem. Nagurney (2003) provides a comprehensive survey of the literature on networks in finance and economics.

It is very natural to use network and graph models to study financial contagion, since banks can be represented by vertices or nodes and the bilateral exposures as edges or arcs in a graph. Thus, it is not by chance that many people have chosen this path for modelling contagion. Building on previous research (Diamond and Dybvig, 1983; Allen and Gale, 1998), Allen and Gale (2000) provide the microeconomic foundations to study financial contagion on two different structures, namely the complete graph and the cycle, which they called a complete and an incomplete market structure respectively. Allen and Gale concluded that the complete structure is more resilient to liquidity shocks than the cycle. Despite their undeniable contribution, the drawback is that real financial networks differ significantly from those two extreme cases, as illustrated by Boss *et al.* (2004a), who give a glimpse of what a real interbank market looks like.

Based on certain characteristics of the model by Eboli (2004), Nier *et al.* (2007) propose a model that captures a more general structure of the financial system. In order to gain insight on financial contagion, the authors randomly generate graphs to simulate interbank markets and then explore the impact of variations in different parameters (e.g. the bank's capitalization) on the possibility of occurrence of bank failures due to contagion. Iori *et al.* (2005) analyse the network of the Italian overnight market and provide some useful measures to characterize the network at different points in time. Additionally, Vivier-Lirimont (2004) studies network structures that would enable banks to improve depositors' utility by means of small-world networks. Small world networks are networks which have a small clustering coefficient and average shortest path length (Watts and Strogatz,

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1998). Such networks have been found to exist in a wide number of social and natural phenomena, like the Internet, genetics, scientific collaboration, etc. The reader is also encouraged to read the articles by Boss *et al.* (2004b) and Muller (2006), who also study financial contagion by means of network or graph models.

Finally, a word is in order on the measurement of contagion; as pointed out by Rigobon (2001), this is not easy to do. There are, however, some interesting proposals in which contagion is measured through equity prices as opposed to banks' balance sheets, like Gropp and Moerman (2004).

3. A NETWORK MODEL OF SYSTEMIC RISK

The following systemic risk model traces losses to the system due to bank failures, whether they are due to the initial stochastic shock or determined by contagion, on a network $G[N,A]$. The nodes N of the graph are partitioned as $N = \{s,S,R,t\}$, where s is the node that represents the initial shock to the system, S is the set of nodes that represents the banks which are the 'sources' of contagion into the system given the initial shock, R is the set of 'relay' nodes which are banks in the possible contagion tree at the different 'stages' of contagion and t represents the sink node where all systemic losses concur. The network is represented schematically in Figure 1.

From Figure 1, it is seen how systemic risk is divided into its two phases, namely the shock phase and the contagion phase. It is also seen that, depending on the phase, the arcs in the network are labelled with different attributes. Thus, the arcs that go from the shock node s to the source nodes are labelled with the individual bank failure probabilities p_i. The labels on the arcs during the contagion phase are the exposures d_{ij} that banks have to each other, which in this model are assumed constant through all possible contagion stages N. Finally, the arcs that go from the terminal relay

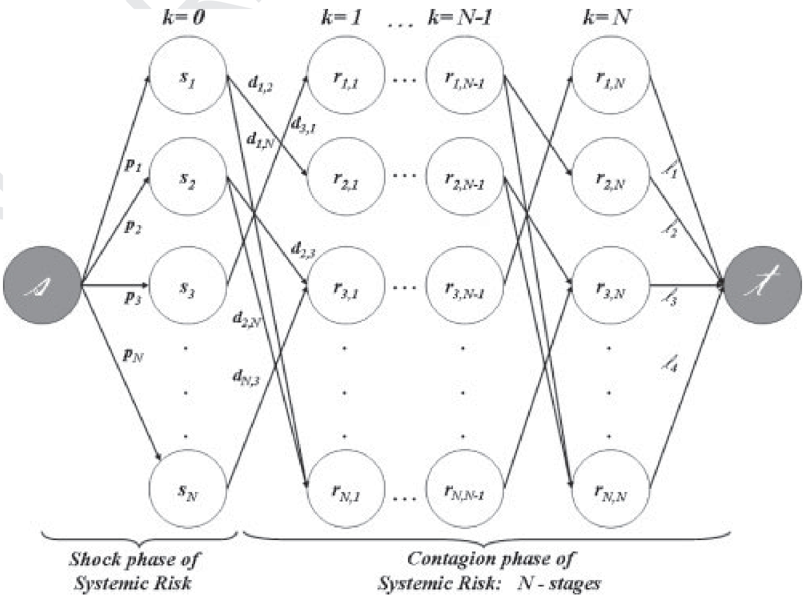

Figure 1. Systemic Risk Network Model

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nodes r_{in} to the sink node t are labelled with the loss to the system l_i given failure of bank i. It should be noticed that, in this simple model, only the initial shock is a random event; the ensuing contagion process is deterministic.

3.1. The Loss Distribution

Let F denote the set of failed banks in the initial shock and $L(F)$ denote the losses to the system if 'scenario F occurs'. Note that L considers both the losses of the banks that fail in the initial shock and the losses due to the contagion generated by these banks. Furthermore, since contagion is a deterministic process, the banks that fail due to contagion of initially failed banks in F is unique; so, let $C(F)$ denote the set of banks that fail due to contagion whose source is F. Then, the loss to the system given scenario F is simply

$$L(F) = \sum_{i \in F} l_i + \sum_{i \in C(F)} l_i \tag{1}$$

Furthermore, assuming that, during the initial shock, the failure probabilities of banks are independent, this loss has probability of occurrence

$$P(F) = \prod_{i \in F} p_i \prod_{i \in F} p_i (1 - p_i)$$

where $F^{\sim}$ is the complement of F.

Thus, doing this for all possible F, the distribution of losses in the system is obtained.

For the sake of clarity, in this simple model it is assumed that bank failures are independent of each other. Although this is a rather strong assumption, it facilitates understanding how the full distribution of losses can be obtained. However, the more realistic case of dependence can be addressed in several ways:

- One can assume that there exist 'implicit correlations' in the default probabilities, i.e. default probabilities are correlated to the extent that they respond to common risk factors to some degree.
- Although complex, it is also possible to derive a formula that contemplates the 'explicit' correlations.
- Finally, it is possible to deal with dependence of joint failures by using a copulas-based approach.

3.2. Discrete Modelling of Contagion in the Network

In order to model contagion, assume that, at every stage of contagion and for each bank i in the system, there is a certain 'threshold' u_i^k such that, if the banks exposure to previously defaulted banks exceeds the threshold, the bank will also fail. Formally, let D^k be the set of all banks that have failed by stage k. Then, bank i will fail at stage $k + 1$ if

$$\sum_{j \in D^{k-1}} d_{ji} \le u_i^k \qquad \text{and} \qquad \sum_{j \in D^k} d_{ji} > u_i^{k+1}$$

We define a state variable to indicate whether a bank is failed or not at stage k of the contagion process as

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$$\theta_i^k = \begin{cases} 1 & \text{if} \quad \sum_{j \in D^k} d_{ji} > u_i^k \\ 0 & \text{otherwise} \end{cases}$$

From here, the modelling of contagion is straightforward:

1. $\sum_i \theta_i^k d_{ij}$ is the sum of defaulted exposures to bank j at stage k
2. $u_j^{k+1} = \max\left\{u_j^k - \sum_i \theta_i^{k-1} d_{ij}; 0\right\}$
3. $\theta_j^k \geq \dfrac{\sum_i \theta_i^{k-1} d_{ij} - u_j^k}{1 + \sum_i \theta_i^{k-1} d_{ij}}$
4. $\theta_j^k < \dfrac{\sum_i \theta_i^{k-1} d_{ij} + \varepsilon}{u_j^k + \varepsilon}$
5. $\theta_j^{k+1} \geq \theta_j^k$ and $\theta_j^k \in \{0,1\}$ $\forall j,k$.

Now, to verify that the above logic will give the state of any bank at every stage, first assume that $\sum_i \theta_i^{k-1} d_{ij} < u_j^k$ so that bank j does *not fail* at stage k. From (3) and (4) above we have that

$$\alpha \leq \theta_j^k < 1 \qquad \text{where} \qquad \alpha = \frac{\sum_i \theta_i^{k-1} d_{ij} - u_j^k}{1 + \sum_i \theta_i^{k-1} d_{ij}} < 0$$

In other words, θ_j^k must be strictly less than one and greater than some negative number. Then, $\theta_j^k = 0$, since θ_j^k can only be zero or one.

Similarly, assume $\sum_i \theta_j^{k-1} d_{ij} > u_j^k$, so that bank j fails at stage k. Again, from (3) and (4) above:

$$0 < \frac{\sum_i \theta_i^{k-1} d_{ij} - u_j^k}{1 + \sum_i \theta_i^{k-1} d_{ij}} \leq \theta_j^k < \beta \qquad \beta = \frac{\sum_i \theta_i^{k-1} d_{ij} + \varepsilon}{u_j^k + \varepsilon} > 1$$

This means that, in this case, θ_j^k must be strictly greater than zero and less than some number which is greater than one. Thus, $\theta_j^k = 1$, since $\theta_j^k \in \{0,1\}$.■

Additionally, we now define a very important concept in the contagion phase: overexposure. We say that a bank i is overexposed if:

$$\sum_{j \in N^-(i)} d_{ji} > u_i^0 \tag{2}$$

where $N^-(i)$ is the set of inner neighbors[4] of bank i.

We can infer from the above definitions that the contagion phase only depends on the set of overexposed nodes and the set of their inner neighbours. This means that, in order to study contagion on a specific network of interbank lending, it is only necessary to focus on the subnetwork that

[4] The set of inner neighbours of node i is the set of all nodes that have a directed arc ending in node i.

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Table I. Probability of default, threshold and loss for banks A, B, C, and D

Bank	Probability (%)	Threshold	Loss
A	1	11	16
B	4	5	20
C	2	7	12
D	1	7	8

consists of overexposed nodes, their inner neighbours and the respective links, since banks that are not overexposed cannot fail during the contagion phase. This simplification of the network considerably reduces the computational effort.

Finally, the losses in the system are computed as

$$L = \sum_i \theta_i^N l_i \tag{3}$$

It is important to notice that L is the sum of losses over all the failed banks at the end of the final contagion round.

3.3. A Toy Example

For illustrative purposes, in this section we give a toy example of systemic risk measurement in a system with only four banks (A, B, C, D) and assume that their probability of default, thresholds and losses given default are as shown in Table I.

Assume their exposures on the interbank market are as shown in Table II. From Table II, we know, for example, that bank A owes 10 units to bank D and the total exposure of bank C is 14 units. Clearly, if none of the banks fail (no shock occurs), then for every i we have $\theta_i^0 = 0$. This means that $D^0 = \varnothing$; therefore, $\sum_{j \in D^0} d_{ji} \leq u_i^0$ for all banks, as can be seen in Tables I and II. Note that, in this system, banks B, C and D are overexposed.

Now assume that bank failure probabilities p_i are independent and as shown in Table I. Examine the case where only bank A fails and is unable to honour its commitments. From Table II, it is seen that only banks B and D are exposed to bank A and that

$$d_{\mathrm{AB}} = 6 > u_{\mathrm{B}} = 5$$

and

$$d_{\mathrm{AD}} = 10 > u_{\mathrm{D}} = 7$$

Table II. Individual bank exposures

	A	B	C	D
A	0	6	0	10
B	0	0	4	8
C	2	0	0	0
D	0	0	10	0
Total exposures	2	6	14	18

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Table III. Loss distribution for the toy problem

Loss	Probability (%)
0	92.12
12	1.88
20	1.88
36	3.96
56	1.00

so that in the first stage of contagion both banks B and D will fail once A has failed. In the next stage, bank C is exposed to banks B and D, so that

$$d_{BC} + d_{DC} = 4 + 10 > u_C = 7$$

Thus, bank C fails in the second stage, so that, if bank 'A' fails in the initial shock, the whole system will fail due to contagion, for a total loss of

$$L = \sum_i \theta_i^N l_i = 56$$

Assuming independence, the probability of this happening is

$$P = p_A(1 - p_B)(1 - p_C)(1 - p_D) = 0.93\%$$

If one repeats the procedure assuming bank B fails in the initial shock, then bank D will fail in the first stage of contagion and bank C will fail in the second. The total loss in this case is $L = 36$ and the probability of this scenario is 3.48%.

Thus, by repeating the procedure for all possible combinations of bank failures during the initial shock (i.e. that one, two, three or all four banks fail in the initial shock) and computing the corresponding losses and probabilities, it is possible to obtain the complete loss distribution. For this simple example, the results are summarized in Table III.

4. A MEASURE OF SYSTEM FRAGILITY

As previously mentioned, it is difficult to measure systemic risk and to assign a risk measure to the contagion process and how fragile a banking system is. Moreover, as in the case of systemic risk, there is little consensus of what financial fragility actually is. For example, Tsomocos (2003) provides the following definition of financial fragility:

> When substantial default of a 'number' of households and banks (i.e. a liquidity 'crisis'), without necessarily becoming bankrupt, occurs and the aggregate profitability of the banking sector decreases significantly (i.e. a banking 'crisis').

Some insight can be obtained from our model. After having experimented with the model, and as will be illustrated in the next section, it appears that, considering the topological aspects of the

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interbank exposures network and the probability distribution of the initial shock, a financial system becomes more fragile when

- there are more overexposed banks;
- there are more paths in the network going through overexposed banks;
- the probability distribution over the shock scenarios weighs more heavily on banks that trigger contagion.

In summary, from our experience with the model, we infer that system fragility is characterized by high default probabilities (initial shock), the associated losses and the propensity to contagion (overexposed nodes). The loss distribution combines all these elements and can be used to derive a fragility measure. For example, the expected loss could be used as a fragility measure, but disregarding the variance of the distribution could be a very misleading appreciation of the actual risk. This observation immediately suggests that a better measure would be related to some quantile α of the distribution or to use VaR(α) directly to derive a measure of fragility for a financial system; for example:

$$\mu = \frac{VaR(\alpha)}{\sum l_i} \tag{4}$$

where the denominator represents the total potential asset losses for the financial system. However, there are many promising candidates for the denominator; for example, the total capital of the banking system.

5. TESTING THE ROBUSTNESS OF THE MEXICAN BANKING SYSTEM

In this section we report the results of applying the model previously described in Section 3 to analyze the robustness/fragility of the Mexican banking system, under normal and stressed conditions. We ran the systemic risk model with information corresponding to the interbank loans reported at the end of 2006 by all the banks in the Mexican financial system. Additionally, since the model is parameterized it is possible to consider different percentages and types of losses; this is a topic in itself. Namely, when a bank fails, different participants lose different things; most notably: Shareholders lose the capital invested, creditors lose what is their due, depositors will lose anything over what is ensured, and ultimately taxpayers will have to pay for the cost of the resolution process. In the literature, the most explored cases are asset losses and the losses due to interbank defaulted loans.[5] Although, there are different percentages of losses reported in the literature, we will only examine the case when the failure of a bank causes a loss that is a percentage of the bank's total assets. The data used in our tests is described in Section 5.1.

We adopt the perspective of a central bank regarding the computation of losses. We consider that the failure of a bank generates a loss to the 'system' (in fact, to the organization in charge of deposit insurance) which is related to the composition of its portfolio. In order to calculate asset deterioration, we consider the historical experience of the Mexican deposit insurance organization on the

[5] For example, see Wells (2002), Blavarg and Nimander (2002), Upper and Worms (2004), Degryse and Nguyen (2004), Graf *et al.* (2005), Boss *et al.* (2004a) Elsinger *et al.* (2006), Boss *et al.* (2004b) and Muller (2006).

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recovery rates from different types of asset, from previous experience in resolution processes in Mexico.

5.1. Data

The central bank has daily data that can be used to calculate the matrix of interbank exposures of the Mexican financial system, from January 2004 onwards. The period of time contemplated in this study goes from this date to December 2006. Although there are 31 banks in the system, the exercises performed only included 25, since the remaining six are relatively new charters for which information is scarce and inconsistent. The interbank exposures considered comprise all the possible deposits, credits and loans, including credit lines as part of the interbank market.[6] For a correct analysis, it is important to know what the real network of exposures looks like and how it changes over time. As it is pointed out in Graf *et al.* (2005), the assumption of maximum entropy on the distribution of the interbank exposures is not realistic, at least not in the Mexican case. Bank failure probabilities for banks in the Mexican financial system are those calculated by the Central Bank. Table IV, shows the individual banks' default probabilities under 'normal' and 'stressed' circumstances, the threshold values and the loss given default. The threshold values and the losses, also

Table IV. Banks' probabilities of failure under normal and stressed circumstances, threshold values and loss given default

Bank	Probability (%)		Threshold	Loss given default
	Normal	Stressed		
B1	0.02	13.91	60 717 019	246 457 752
B2	0.02	16.18	55 315 788	99 651 958
B3	0.02	22.60	35 585 033	153 017 549
B4	0.17	24.63	18 679 785	98 224 226
B5	1.37	0.01	725 260	5 428 626
B6	0.02	0.01	3 328 200	8 545 224
B7	0.02	0.72	776 791	4 554 604
B8	0.15	21.49	17 393 247	18 995 761
B9	1.24	35.84	848 020	1 618 274
B10	0.02	16.62	728 732	2 211 619
B11	0.02	14.12	14 154 915	40 186 211
B12	0.02	14.67	818 166	1 848 334
B13	0.02	15.90	1 008 479	4 061 737
B14	0.02	0.68	604 450	480 709
B15	0.11	0.81	638 665	1 267 252
B16	0.02	32.88	23 684 692	35 094 441
B17	0.10	0.02	555 356	2 188 063
B18	0.02	0.02	1 201 305	7 066 934
B19	0.02	0.05	1 821 349	8 623 238
B20	0.02	0.02	488 302	553 387
B21	0.02	0.02	3 461 918	18 570 612
B22	0.02	1.26	519 252	1 180 286
B23	1.16	0.37	2 041 173	27 718 638
B24	0.02	0.01	281 461	87 654
B25	0.02	0.02	1 244 944	2 549 072

[6] We are currently working with an extended set of exposures for the interbank market and the number of loans and its total volume are indeed larger.

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Table V. Statistics for the normal and stressed probabilities

	Probability (%)	
	Normal	Stressed
Average	0.18	9.31
Standard deviation	0.40	11.5
Maximum	1.36	35.84
Minimum	0.01	0.01

shown in Table IV, will be used in the four reported experiments. The only sources of 'stress' will be the default probabilities and the network topology.

Table V shows some basic statistics of the two (previously defined in Table IV) different sets of bank failure probabilities, i.e. the reference case and the stressed case, where one can see that the latter are much larger than the former.

The interbank lending market plays a crucial role in liquidity transmission between banks. This market is the most recognized (or at least one of the most studied) channels for financial contagion. Table VI shows the Mexican interbank lending market as it was on the 29 December 2006.[7] From Tables IV and VI we can infer that there is only one 'overexposed' bank, namely B15. This bank's exposures were 590 139 to B1, 510 000 to B6 and 135 000 to B12, which add up to 1 235 139; the threshold value for this bank is 638 665, which is clearly exceeded by its exposures. Generally speaking, the Mexican interbank market during the period of study presents very few 'overexposed' banks.[8] It is important to stress that the interbank exposures considered in this article are the gross exposures as opposed to the net exposures, as is common in this type of research. If net exposures are considered instead of gross exposures then we would be reducing the exposures on the interbank market, which in turn could cause an underestimation of the effect of contagion.

To clarify this issue we will elaborate an example. Let us assume that bank A lends \$50 to bank B and that bank B is lending \$20 to bank A, the net exposure is equivalent to a \$30 loan from bank A to bank B. If bank B fails, then bank A will be instantaneously deprived from its \$50 loan, this amount of money will be compromised until the situation of bank B is clarified and the resolution process takes place, which could be a very long and painful process. The most important fact is that bank A might be a healthy institution but during the resolution process of bank B, bank A cannot count on bank B to honor its debt. On the other hand, if bank A fails, then bank B still has lent \$20 to bank A and bank B will not be receiving this money immediately and this could threaten the stability of bank B, regardless of the net exposure.

Table VII shows the maximum historic exposures registered during the period between January 2004 and December 2006.[9] Contrary to what we saw in Table VI, we have that almost all the banks are 'overexposed', and this could result in financial fragility of the system due to contagious defaults. From Table VII we can infer that there have been exposures between almost every pair of banks.

[7] Quantities are in millions of pesos for presentation purposes.
[8] Never more than three. However, with a more inclusive approach regarding the components of the interbank loans, this could change dramatically.
[9] Quantities are in millions of pesos for presentation purposes.

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Table VI. Interbank market exposures on the 29 December 2006

Bank	B1	B2	B3	B4	B5	B6	B7	B8	B9	B10	B11	B12	B13	B14	B15	B16	B17	B18	B19	B20	B21	B22	B23	B24	B25
B1	0	3451	9391	4112	0	1034	0	661	24	119	79	8	0	197	590	369	259	0	548	0	0	0	0	0	0
B2	0	0	0	0	80	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
B3	0	0	0	1400	0	0	0	0	0	0	0	0	0	0	0	0	0	0	391	0	0	0	0	0	0
B4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	100	0	0	0	0	0
B5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
B6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	510	0	0	0	0	0	0	0	202	0	0
B7	0	0	0	0	0	54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
B8	1000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
B9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
B10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
B11	0	0	0	0	0	400	0	0	0	0	0	0	137	0	0	0	0	57	0	0	0	216	0	0	0
B12	0	0	0	0	0	0	0	0	0	0	0	0	0	27	135	0	0	0	0	0	0	0	0	45	0
B13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
B14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
B15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
B16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
B17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
B18	578	0	0	0	0	0	0	0	0	0	0	0	0	25	0	0	0	0	0	0	0	0	0	0	0
B19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
B20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
B21	0	0	0	0	0	0	0	0	0	0	162	0	0	0	0	0	0	0	0	27	0	0	0	0	0
B22	0	0	0	0	0	0	22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
B23	0	0	0	0	0	106	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
B24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
B25	209	151	0	302	0	0	0	0	50	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

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Table VII. Interbank market's maximum historic values

Bank	B1	B2	B3	B4	B5	B6	B7	B8	B9	B10	B11	B12	B13	B14	B15	B16	B17	B18	B19	B20	B21	B22	B23	B24	B25
B1	2000	172	310	600	590	5653	2809	569	3214	325	1840	0	0	0	0	5653	2809	569	3214	325	1840	0	0	0	0
B2	2000	145	1281	135	600	5200	2569	198	10794	359	2352	316	529	1	37	5200	2569	198	10794	359	2352	316	529	1	37
B3	1004	129	565	800	700	4700	4338	130	2455	440	1196	144	1000	54	0	4700	4338	130	2455	440	1196	144	1000	54	0
B4	500	0	376	0	0	1913	59	0	500	215	1500	0	0	0	0	1913	59	0	500	215	1500	0	0	0	0
B5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
B6	0	150	215	351	626	200	0	150	0	0	747	160	600	60	0	200	0	150	0	0	747	160	600	60	0
B7	50	64	79	75	120	78	0	0	0	0	0	55	90	0	0	78	0	0	0	0	0	55	90	0	0
B8	1000	0	10	300	700	1000	370	0	36	0	811	44	59	0	0	1000	370	0	36	0	811	44	59	0	0
B9	0	0	45	51	195	100	0	161	0	0	0	71	32	0	0	100	0	161	0	0	0	71	32	0	0
B10	10	109	224	48	200	0	0	70	0	0	0	43	86	63	0	0	0	70	0	0	0	43	86	63	0
B11	0	212	439	230	600	1000	0	179	141	200	887	216	500	46	0	1000	0	179	141	200	887	216	500	46	0
B12	45	0	125	100	240	31	0	77	0	0	126	60	0	93	0	31	0	77	0	0	126	60	0	93	0
B13	100	100	0	18	250	0	0	47	0	0	0	25	10	0	0	0	0	47	0	0	0	25	10	0	0
B14	0	117	60	0	205	0	0	50	0	0	0	5	115	0	0	0	0	50	0	0	0	5	115	0	0
B15	0	134	183	205	0	205	0	68	0	0	0	136	250	0	0	205	0	68	0	0	0	136	250	0	0
B16	802	218	519	1000	600	0	280	122	838	420	1841	299	307	0	0	0	280	122	838	420	1841	299	307	0	0
B17	0	0	0	0	0	30	0	0	0	0	100	0	0	0	0	30	0	0	0	0	100	0	0	0	0
B18	4	109	200	100	350	0	0	0	0	29	50	55	68	10	0	0	0	0	0	29	50	55	68	10	0
B19	150	0	0	0	0	110	50	0	0	0	430	0	0	0	0	110	50	0	0	0	430	0	0	0	0
B20	0	0	0	95	0	25	0	52	40	0	154	157	0	0	0	25	0	52	40	0	154	157	0	0	0
B21	850	70	0	0	160	1000	130	44	220	355	0	59	0	0	0	1000	130	44	220	355	0	59	0	0	0
B22	0	39	35	49	87	0	0	70	0	0	0	0	20	0	0	0	0	70	0	0	0	0	20	0	0
B23	0	0	156	186	366	180	0	212	0	0	0	92	0	80	0	180	0	212	0	0	0	92	0	80	0
B24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
B25	0	0	250	0	0	101	0	0	0	0	0	82	0	0	0	101	0	0	0	0	0	82	0	0	0

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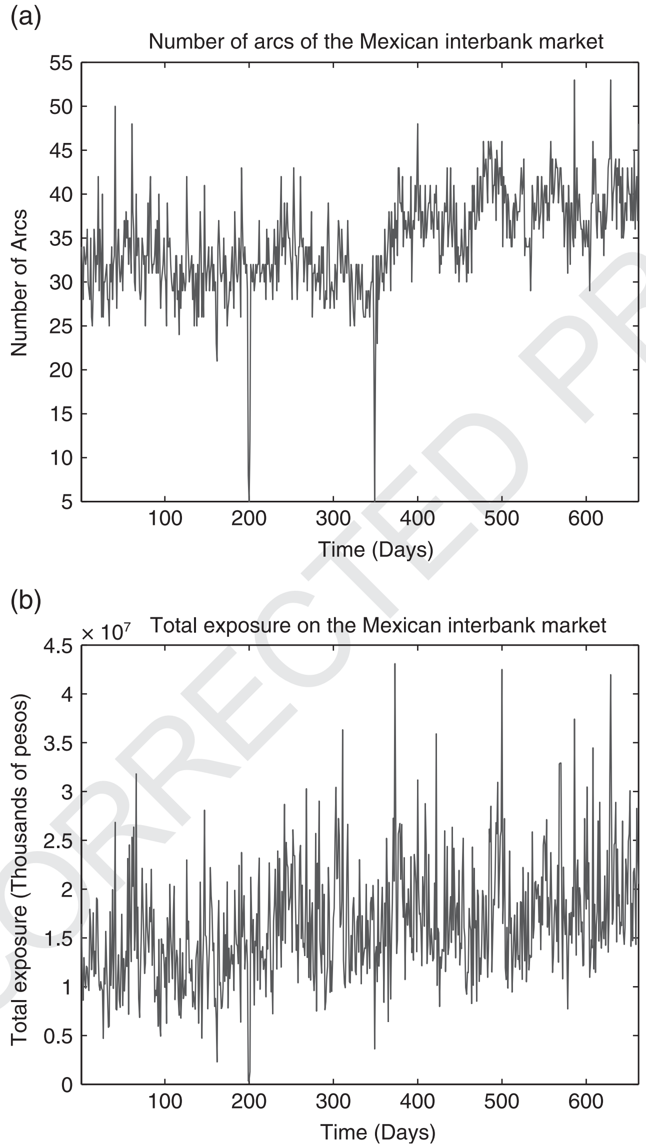

Figure 2. Evolution of the interbank market: (a) total number of arcs (exposures); (b) total exposure on the interbank market

However, such relationships do not remain constant over time, as most of the time the market behaves more like the market on 29 December 2006.

Figure 2 shows the evolution through time of two important aspects of the interbank market: first, we have in Figure 2a the evolution of the number of arcs of the interbank market network; second, we have in Figure (b the total exposure of the financial system in the interbank market. From Figure 2a we can see that the number of exposures is usually between 25 and 50 with two extreme falls in

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the number of loans which are associated with two large adjustments in the BMV's IPC.[10] In Figure 2b we can see an upward trend in the total exposure of the interbank market and we can see a large drop around day 200 that corresponds to one of the previously mentioned IPC falls.

5.2. Computational Aspects

At first sight, to compute all the possible shock scenarios and contagion paths appears to be a formidable task. Since there are 25 banks included in the model, there are 2^{25} different combinations of failures due to the initial shock, to which one must add the computation of all the ensuing contagion trees. In the case of the Mexican banking system, however, it is a relatively easy task. Since the only relevant banks in the contagion process are those that are overexposed, and resorting to some of the techniques commonly used in the constraint satisfaction field and implicit enumeration, it is possible to program the algorithm to run in a few hours.[11] In the context of our problem, and referring to our toy example, note that the failure of bank A leads to the complete breakdown of the system; therefore, any combination that includes the failure of bank A is going to cause the system's collapse. Thus, it is not necessary to enumerate all the combinations explicitly, since the outcome is known beforehand. Similarly, since no bank that is underexposed can fail due to contagion, eliminating them from the contagion network also reduces the search space; the more of these that there are, the less cases that have to be explicitly enumerated; such performance enhancement is inspired on the domain reduction technique. For a good introduction to the various techniques available in this field, please consult Tsang (1993).

For example, assume that bank A fails due to the initial shock. Using the domain reduction technique we will only focus on the outer neighbours of A (the banks which lend to A on the interbank market) that are overexposed. This reduces enormously the possible number of contagion verifications by the algorithm. Another example is inspired by the learning no goods technique and consists in keeping in a set the bank failures which cause other contagious failures, i.e. assume that bank A fails due to the initial shock and bank B fails as a consequence. In all the possible subsequent combinations of initial failures (which are many) in which bank A is considered, bank B will certainly fail and we make it fail without executing the costly contagion verification procedure. Another valuable and simple shortcut used was to precalculate the probability of failure of the failing subset before launching the contagion verification procedure. Thus, if one of the participants had zero probability of failure, then this means that the probability of failure of the whole failing subset is zero. By so doing, the expensive contagion verification procedure can be avoided.

To conclude: despite the enormous number of combinations of joint failures which are possible in a small size banking system, it is possible to perform the full enumeration process and estimate the distribution of losses by incorporating some ideas from the constraint satisfaction field and by implementing some simple yet useful shortcuts. Nevertheless, using all the experience obtained during the implementation of this algorithm, we decided that by using simulation we could achieve similar results with much less computational effort. The proposed new approach (under development) is explained in a schematic fashion in Section 6.

[10] The IPC is the main index of the Mexican stock market: the Bolsa Mexicana de Valores (BMV).

[11] Constraint satisfaction techniques (e.g. constraint propagation, domain reduction and learning no goods) are used to reduce the solution search space. Thus, a large number of cases are enumerated implicitly and not explicitly, greatly reducing the necessary computation.

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5.3. Reference Case and Stress Test Results

In order to illustrate the use of the model, we now present the results obtained in four different cases. The first case can be considered as the reference case, and the other three are stressed scenarios. In posing stress scenarios for the financial system it must be recalled that system fragility has to do with bank failure probabilities, the number and importance of overexposed banks, and the number of paths that go through them. Thus, a stress situation has two distinct elements: the bank failure probabilities in the initial shock phase and the interbank exposures in the contagion phase. The reference case is the analysis of the Mexican banking system under current 'normal' conditions. In stress case 1, the interbank exposures are stressed while maintaining bank failure probabilities as in the reference case. In stress case 2, interbank exposures are as in the reference case and bank failure probabilities are stressed. Finally, in stress case 3, both failure probabilities and interbank exposures are stressed. Specifically:

- *Reference case* For this case, the interbank exposures are taken as those observed at the end of December 2006. The failure threshold values are taken as tier 1 capital at the end of December 2006. Banks' failure probabilities are estimated from market and credit risk data over the period 2001–2006. These default probabilities can be considered as the probabilities under 'normal' conditions, since the 2001–2006 horizon does not include periods of crises. Finally, the losses are taken as different percentages of each bank's total assets in December 2006. The individual percentages of losses used for this study are derived from the historic experience of the IPAB[12] on the recoveries (and losses) for different types of asset. Owing to the different composition of the assets for each bank, the resulting percentages are unique and can vary widely from 11% to 83%.
- *Stress case 1* For this case, the interbank exposures are taken as the maximum registered historic values, the rationale being it was our intention to investigate what would happen in a network that possesses a large number of links and overexposed banks. The other parameters are the same as for the reference case.
- *Stress case 2* For this case, interbank exposures and thresholds are the same as for the reference case, whereas banks' failure probabilities are estimated considering the period 1994–2001, where the Mexican banking system went through several critical periods. The stressed probabilities were calculated so as to characterize a period of extreme financial distress for the banking system, such as the Mexican 1994 crisis. As done previously, losses are taken as a percentage of the banks' total assets in December 2006.
- *Stress case 3* For this case, interbank exposures are as in stress case 1 and failure probabilities are as in stress case 2. As usual, the losses are taken as a percentage of the banks' total assets in December 2006. This case is obviously the most dramatic one, as the network contains a large number of links and overexposed banks and the failure probabilities are those of the stress period.

Figure 3 shows the two different graphs representing interbank exposures. Figure 3a shows the reference case, which was the state of the interbank market at the end of December 2006. Note that there is only one overexposed bank (represented by the red circle). The stressed graph (Figure 3b) shows the maximum historic exposures between banks, where almost every bank is overexposed. 3

[12] Instituto de Proteccíon al Ahorro Bancario: the Mexican deposit insurance institution.

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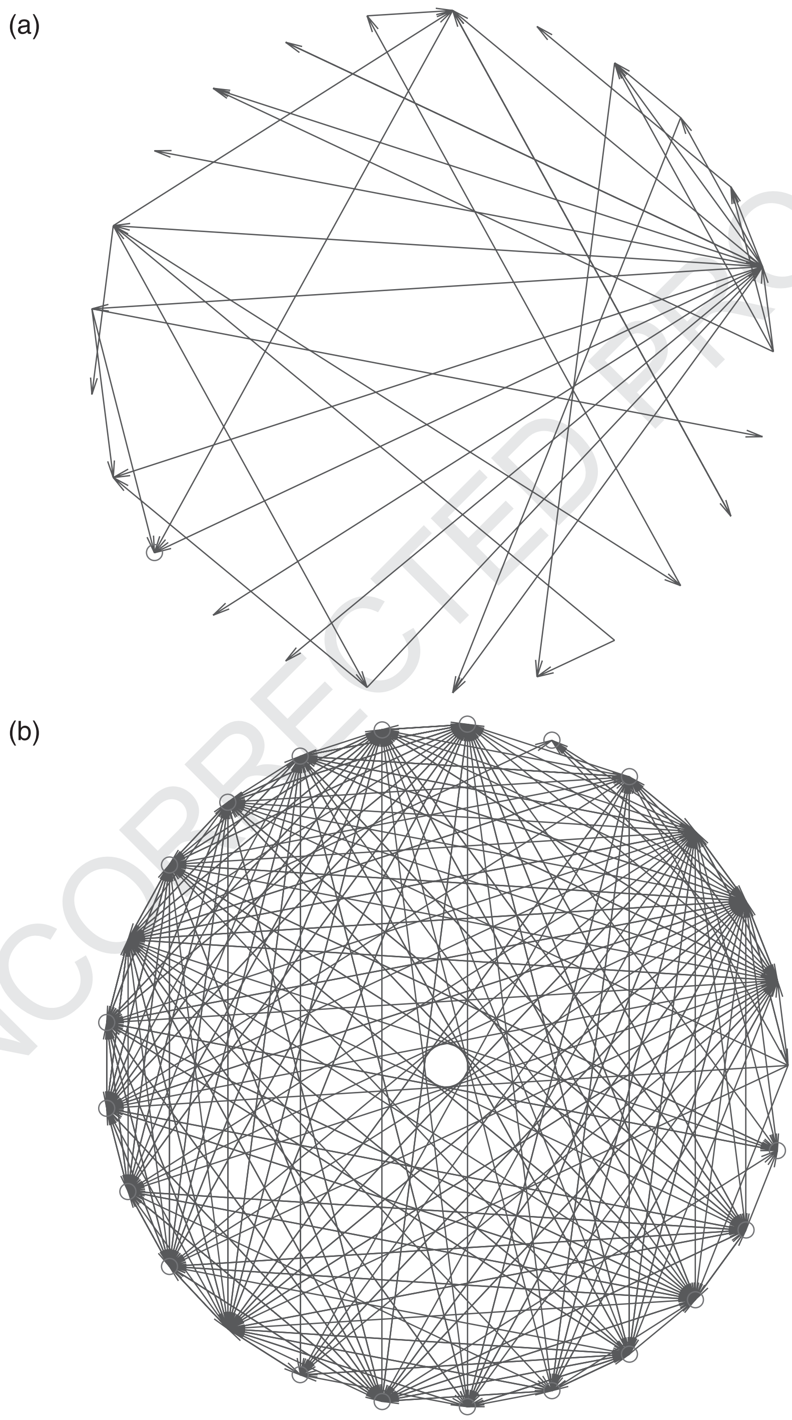

Figure 3. Two different exposures networks: (a) December 2006; (b) maximum historic loans

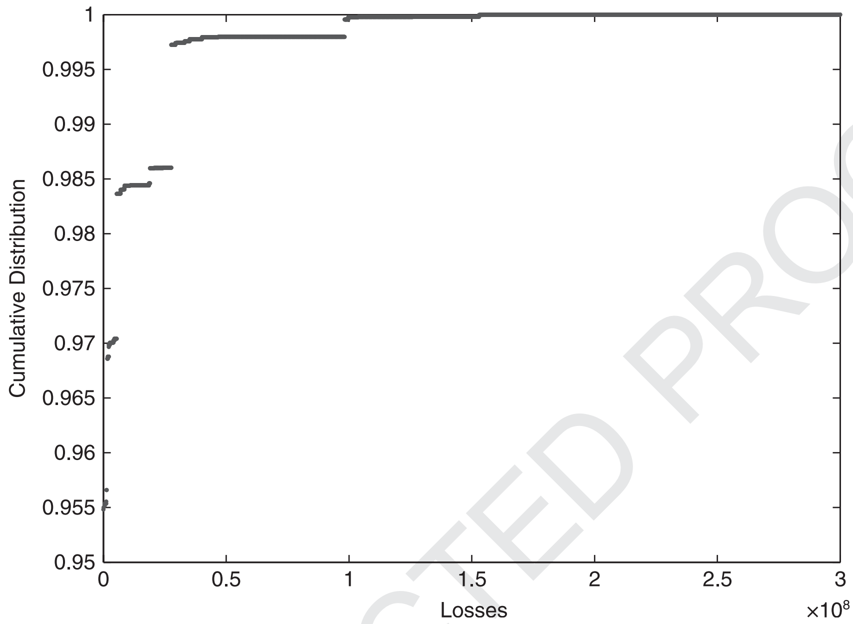

Figure 4. Distribution of losses for reference case

Figure 4 shows the cumulative distribution of losses for the reference case, where we see that there is a very high probability of losing nothing and there are big jumps for small losses. As we move along the *x*-axis, it is seen that the probabilities of large losses are very small. We can say that this is a typical example of a distribution of losses for a financial system where the probability of individual failures of banks is generally very small and the probabilities of joint failures (the ones that would carry large losses) are even smaller.[13] This is further enforced by the fact that there is practically no possibility of contagion.

Figure 5 shows the distribution of losses for stress case 1, which does not change very much with respect to the reference case. This means that, despite the network topology being dramatically different, the shape of the distribution does not change much. In fact, as we will see later, the VaR obtained is the same for both cases.

Figure 6 shows the distribution of losses for stress case 2, where we see that the distribution of losses changes dramatically in relation to the reference case and stress case 1. In fact, the shape of the distribution is totally different to the two previous cases. Although we acknowledge that we are using 'extreme' probabilities, it is remarkable that a drastic change in topology of the interbank exposure network has such a small effect on the distribution of losses, whereas the opposite is true when one changes the failure probabilities.

[13] For example: a typical probability of an individual bank failure is 0.02%. The probability of a simultaneous failure of 16 banks that individually have a 0.02% of default probability would be (assuming independence) in the order of 0.0216, which is a very small number.

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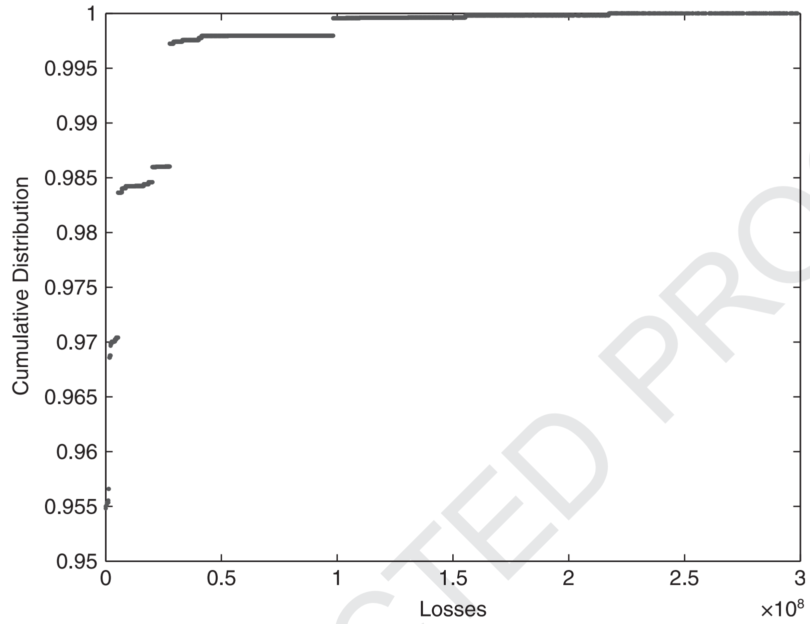

Figure 5. Distribution of losses for stress case 1

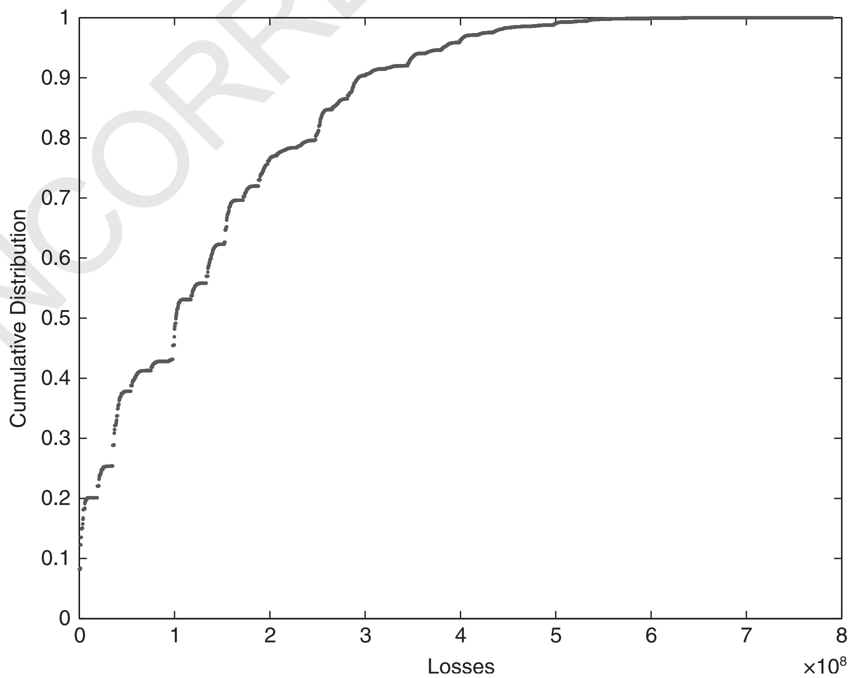

Figure 6. Distribution of losses for stress case 2

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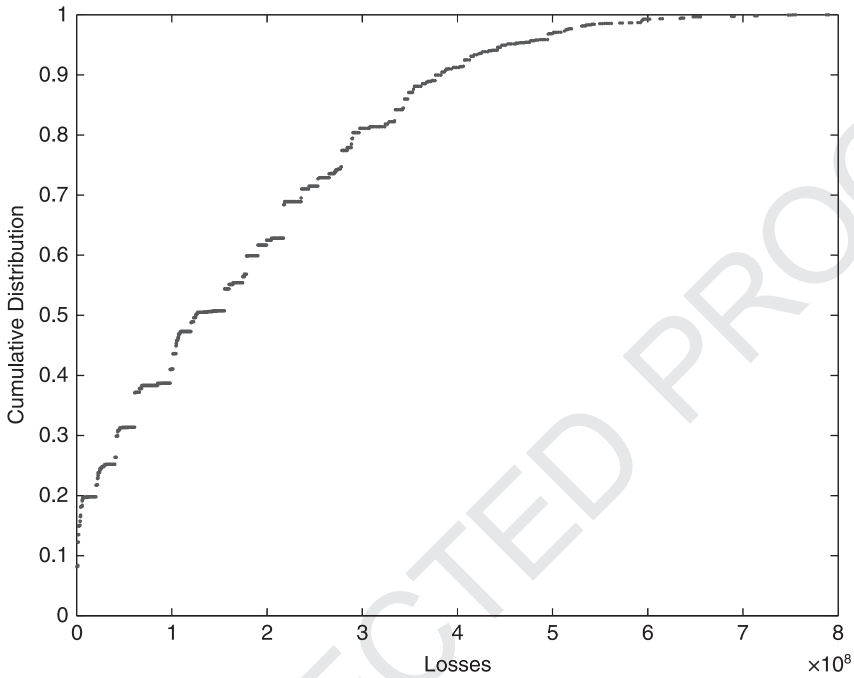

Figure 7. Distribution of losses for stress case 3

Figure 7 shows the distribution of losses for stress case 3, where we see that the distribution of losses does change dramatically in relation to the reference case and stress case 1 but less in relation to stress case 2. This reinforces the fact that system fragility is more sensitive to failure probabilities than it is to interbank exposures.

Table VIII provides a statistical summary for the distributions of losses for the four cases presented: mean, variance, skewness, kurtosis, VaR(99), the mean divided by the total losses L and the VaR(99) divided by the total losses L. In the table, we see that although the reference case and stress case 1 have the same VaR(99) value,[14] the other statistics of the distributions are

Table VIII. Summary statistics for the loss distributions

	Reference case	Stress case 1	Stress case 2	Stress case 3
Mean	683 431	710 966	129 744 000	167 476 000
Standard deviation	5 753 762	6 371 700	122 875 000	153 211 000
Skewness	14.88	18.05	1.03	0.85
Kurtosis	295.71	457.42	3.60	3.15
VaR	27 718 638	27 718 638	499 317 801	594 429 329
Mean/L (%)	0.09	0.09	16.42	21.19
VaR/L (%)	3.50	3.50	63.19	75.22

[14] It is important to note that, although counterintuitive, this fact can be easily verified by inspecting Figures 4 and 5, in which we see a big jump precisely around the 99% quantile. Since the quantile falls in the gap, both distributions necessarily have the same VaR(99). Moreover, we acknowledge that this is one of the main criticisms of the VaR measure.

significantly different, which means that the shapes of the distributions do matter. Obviously, the reference case and stress case 1 are very different from the remaining two cases, being much less critical. The most important inference that we can make based on the previous results is that, apparently, the loss distribution is much more sensitive to failure probabilities than to the interbank exposure network. It is also interesting to see that the VaR/Total loss measure behaves well as a measure of system fragility.

6. CONCLUSIONS

The most important conclusions that we can extract from our work are as follows. First, although the proposed network model to study systemic risk is very simple, it captures the essential elements to analyse systemic risk in its two main components, namely the initial shock and the ensuing financial contagion. Next, for banking systems with relatively few banks, such as the case of Mexico, it is possible to estimate the distribution of losses for the financial system by total enumeration using efficient computational tools. In larger banking systems one would probably have to resort to Monte Carlo simulation. The model allows the researcher to investigate different aspects of systemic distress. We illustrated the model's flexibility by computing the distribution of losses in four cases of varying conditions of stress. Since the model is totally parameterized, it is a simple matter to study the system by changing any of the parameters. For example, a stress test could be performed by varying the banks' failure thresholds as well as the other characteristics. It can also be used to determine the losses to different participants, i.e. banks' shareholders, creditors, depositors and the taxpayers, as previously pointed out. In fact, all of these types of loss can be estimated simultaneously. Another important finding is that the banking system's fragility is determined by bank default probabilities and the overexposed banks in the network. Also, the banking system is apparently more sensitive to default probabilities than to network structure (overexposed banks).

We are currently working on the relaxation of the independence of bank failures assumption, by including explicit failure correlations and using a copula approach in the estimation of the loss distribution. Figure 8 describes schematically how the simulation algorithm works.

Simulation became necessary due to the increase in the number of banks in the Mexican banking system from 29 to 41. At the present time it is impossible for us to use the same enumeration process despite the use of constraint satisfaction techniques. Finally, since contagion comes in many forms, we believe that more research needs to be done in order to model losses due to contagion in a more realistic way.

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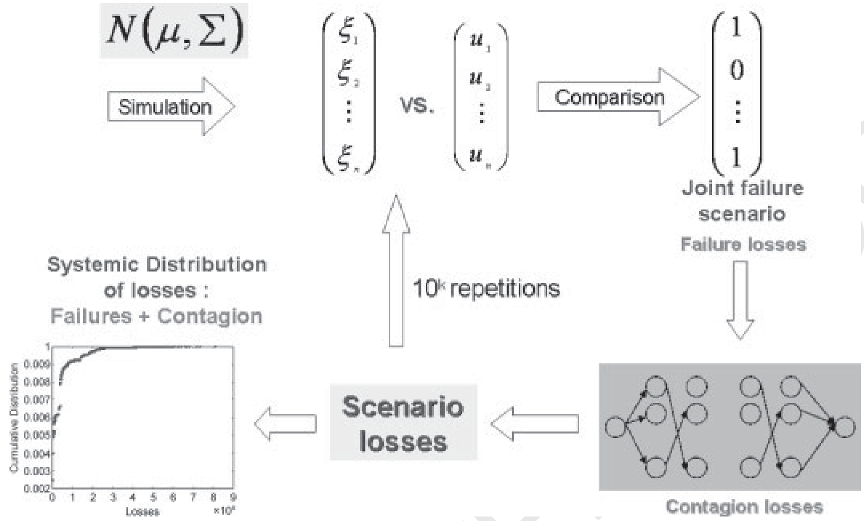

Figure 8. The simulation algorithm

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